NUMERICAL COMPUTING BACKPAPER EXAM

Date: 6th June, 2025, Time: 10 am - 1 pm, Total Marks: 100

Attempt ANY FIVE questions

(1) Describe the complete Horner's algorithm. Using the complete Horner's algorithm, find the Taylor series of the function $f(x) = 3x^5 - 2x^4 + 15x^3 + 13x^2 - 12x - 5$ at the point $x = 2$. (10 + 10 = 20 marks)

(2) Describe Gaussian elimination with scaled partial pivoting. Carry out Gaussian elimination with scaled partial pivoting on the 4×4 matrix $\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 3 & -1 \\ 3 & -3 & 0 & 6 \\ 0 & 2 & 4 & -6 \end{bmatrix}$ and show all intermediate matrices. (10 + 10 = 20 marks)

(3) Desrcribe Newton's method to find roots of a differentiable equation. Prove that two of the four roots of $x^4 + 2x^3 - 7x^2 + 3 = 0$ are postive. Find the approximate values of these roots by using Newton's method. For each root, do two iterations of Newton's method. (10 + 10 = 20 marks)

(4) What is Newton algorithm for finding an interpolating polynomial to fit given data? You are given the following data about a function f: $f(0) = 1, f(1) = 9, f(2) = 23, f(4) = 93, f(6) = 259$. Construct the divided difference table for this data and using Newton's interpolation polynomial, find an approximation to $f(4.2)$. (10 + 10 = 20 marks)

(5) Describe the composite Trapezoid Rule. What is the formula you get by using the composite trapezoid rule on $f(x) = x^2$, with interval $[0, 1]$ and $n + 1$ equally spaced points with $x_0 = 0$ and $x_n = 1$? Simplify your result using the formula for the sum of squares of the first n positive integers. What does this trapezoid estimate converge to as $n \to \infty$? (20 marks)

(6) Describe the method of natural cubic spline (used for interpolation). Find a cubic spline S over knots $-1, 0, 1$ such that $S''(-1) = S''(1) = 0$, $S(-1) = S(1) = 0$ and $S(0) = 1$. (10 + 10 = 20 marks)

(7) Describe the second order Runge Kutta method used to solve an ordinary differetial equation (IVP). Solve the differential equation $\frac{dx}{dt} = -tx^2, x(0) = 2$ at $t = 0.2$, correct to two decimal places, using one step of Taylor series method of order 2 and one step of the Runge-Kutta method of order 2. (Help with formulae: The second order Runge-Kutta method is: $x(t + h) = x(t) + \frac{1}{2}(K_1 + K_2)$, where $K_1 = hf(t, x)$ and $K_2 = hf(t + h, x + K_1)$). (10 + 10 = 20 marks)

(8) Prove that a real square matrix on which naive Gaussian elimination process can be done, can be written as a product of a unit lower triangular matirx and an upper triangular matrix. Is this decomposition unique? Show that the 3×3 matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix}$ cannot be factored into a product of a unit lower triangular matrix and an upper triangular matrix. Interchange the rows of this matrix so that such a factorization can be done. (10 + 10 = 20 marks)